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Instrumentation Analysis Using Parameter Estimation of Simulated Data

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Introduction

PARAMETER estimation is an important tool in the development of aerodynamic models for aircraft. By processing data recorded during flight tests, parameter estimation techniques can calculate estimates of the aircraft stability and control derivatives. Software designed to perform parameter estimation is typically tested on simulated data created from a known set of parameters before it is used to examine actual flight data. This research focuses on expanding the role of simulated data beyond estimator validation to examine a wide variety of parameter estimation issues. The goal of the project is to improve the effectiveness of aircraft model development from flight test data.

A previous paper described an analysis package developed to assist with the preliminary analysis of maneuver design, flight conditions, instrumentation, data processing, and model structure.[1] For this research, the analysis package was used to examine the instrumentation system of one of North Carolina State University's unmanned air vehicles. To provide realistic simulated data, the aircraft was modeled using an existing simulation environment.[2] The simulation includes an elaborate model of the instrumentation system, incorporating the effects of sensor position, sensor dynamics, random noise, analog filtering, and digital storage for each transducer in the system.

The analysis package employs the output error approach to estimate the parameters in a state space model of the aircraft dynamics. Output error has been used for aircraft analysis more than any other parameter estimation technique.[3,4] There are a number of newer and more advanced techniques available now,[5–7] but output error was chosen because of its proven track record for consistent and reliable results. The analysis package was used to study a wide variety of instrumentation issues, but the most significant results involve the analysis of the angle of attack transducer.

Alpha Vane Transducer

The angle of attack (α) measurement is one of the primary observations for an aircraft longitudinal dynamic model. The unmanned air vehicle measures the angle of attack with an alpha vane mounted on a boom in front of the aircraft. The dynamic response of the vane is dependent on the vane sizing, the moment of inertia, and the friction of the system. The response of the vane can be modeled as a second-order low-pass filter of the form

$$G_{\text{alpha}}(s) = \frac{\omega_n^2}{s^2 + 2\omega_n \zeta s + \omega_n^2}$$

The natural frequency ω_n and damping ratio ζ of the vane are estimated to be 5 Hz and 0.7, respectively. In the simulation the continuous transfer function is approximated as a second-order difference equation.

Preliminary Results

The first experiments with the analysis tools attempted to obtain a simple longitudinal model of the aircraft from a set of data recorded on the flight simulator. The results were disappointing. Table 1 shows the estimates and errors for the four main parameters. Large errors in the critical parameter $C_{N\alpha}$ were observed for a wide variety of input maneuvers. To identify the problem, different portions of the instrumentation model were deactivated in the flight simulator. The data collected without the sensor dynamic models showed a noticeable improvement in the estimates. Further analysis identified that the alpha vane was responsible for the inaccuracies, because the other longitudinal sensors (pitch rate and vertical accelerometer) have very fast response times. The lag introduced by the alpha sensor dynamic model causes a delay in the data that interferes with the output error calculation of the stability derivatives.

Several corrections for this problem were considered. A faster approximation of the vane dynamics could be used, but it might be impractical to construct a transducer to match these dynamics. Alternatively, the output error model could be modified to include the dynamics of the vane as states, but this would increase the model complexity and potentially degrade the performance of the estimator. The most desirable correction would be to remove the lag from the angle of attack signal. An effort was made to perform this correction in the postprocessing of the recorded data, but the relatively slow sampling rate (25 Hz) makes this impractical. The remaining option was to design an analog lead circuit for the instrumentation system to modify the signal before it is digitally sampled and recorded by the flight computer.

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Table 1 Parameter estimation results

	Without lead circuit		With lead ($\omega_n = 5$ Hz)		With lead ($\omega_n = 3$ Hz)	
Parameter	Value	Error, %	Value	Error, %	Value	Error, %
C_{N_α} (6.36)	6.554	3.048	6.455	1.488	6.418	0.914
C_{m_α} (−0.367)	−0.3692	0.560	−0.359	2.298	−0.354	3.494
C_{m_q} (−21.6)	−19.162	11.287	−19.401	10.179	−19.699	8.800
$C_{m_{\delta e}}$ (−1.82)	−1.743	4.209	−1.745	4.129	−1.755	3.573

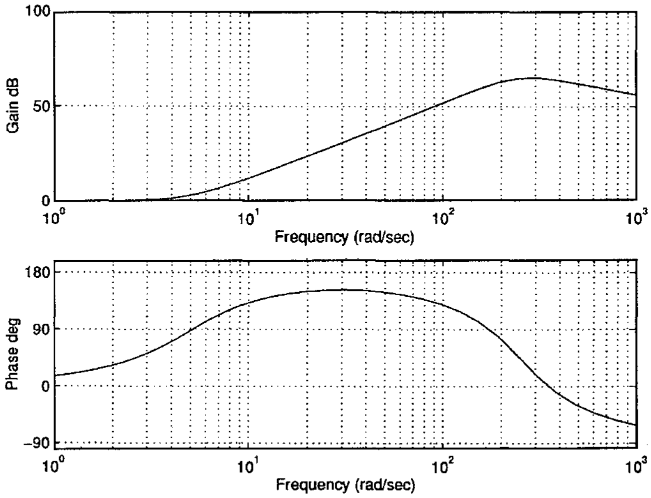

Fig. 1 Bode plot of lead circuit transfer function.

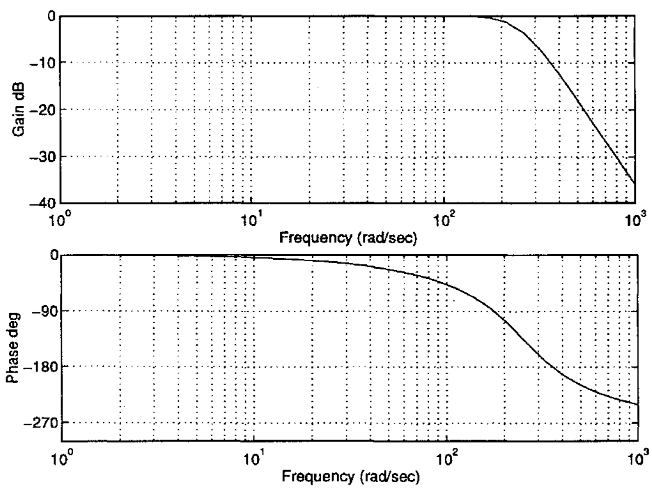

Fig. 2 Bode plot of combined sensor and lead circuit transfer function.

Lead Circuit Design

The lead circuit is designed to cancel the lag effect of the transducer transfer function. A third-order Butterworth filter is used with the following transfer function:

$$G_{\text{Lead}}(s) = \frac{(80\pi)^3\left(s^2 + 2\omega_{n_L}\zeta_L s + \omega_{n_L}^2\right)}{\omega_{n_L}[s + 40\pi(1+\sqrt{3}i)][s + 40\pi(1-\sqrt{3}i)](s + 80\pi)}$$

where ω_{n_L} and ζ_L are the estimated natural frequency and damping ratio of the alpha vane. The numerator of this function is designed to cancel as closely as possible with the poles of the vane dynamics. The three poles of the filter provide a minimal ripple effect on the output of the transducer. This type of filter is optimally flat in the pass band. Figure 1 shows a Bode plot of the transfer function for the lead circuit. Figure 2 shows the Bode plot for the combination of the sensor and lead circuit. For low frequencies the phase shift is negligible. The simulator sensor module incorporates a discrete representation of the lead circuit operating at the simulation frequency of 100 Hz. This allows the 40-Hz dynamics of the Butterworth filter to be adequately represented.

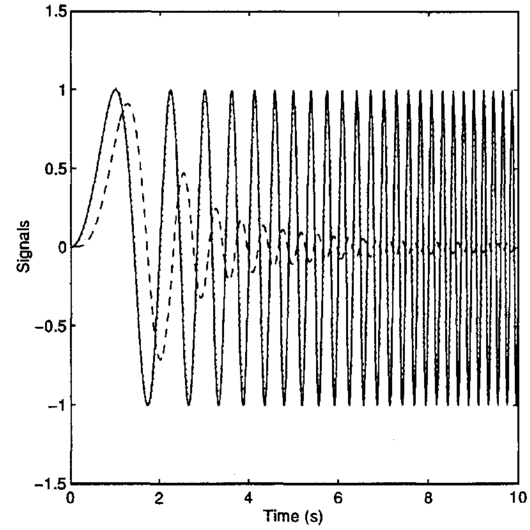

Fig. 3 Signal response for the alpha sensor and lead circuit: ——, input; – – –, sensor out; · · · ·, lead out.

Results with Lead Circuit

The theoretical performance of the lead circuit for a sinusoidal frequency sweep is shown in Fig. 3. The true signal, sensor output, and lead circuit output are plotted. The lead circuit clearly improves the magnitude and time response of the system. The parameter estimation results for data collected using the lead circuit are also considerably improved. Table 1 shows that the average error for the $C_{N\alpha}$ estimate was reduced by nearly 50%.

Lead Circuit Robustness

The lead circuit used in the example was designed with the exact natural frequency and damping ratio of the alpha vane, so the pole-zero cancellation is nearly perfect. For the actual circuit design, the exact model of the vane will not be available and the analog filter components will not provide a perfect filter. To ensure that the filter will be effective at these nonideal conditions, the robustness of the design was examined. Using the same lead circuit design frequency ($\omega_{n_L} = 5$ Hz), the alpha vane model was changed to a slower system ($\omega_n = 3$ Hz). Table 1 confirms that there was no noticeable reduction in the accuracy of the parameter estimates.

Conclusions

The preliminary analysis of the simulated instrumentation system indicated that the slow dynamics of the alpha vane transducer have an adverse effect on the quality of the data obtained from the parameter estimation technique. An analog lead circuit was developed and modeled in the simulated instrumentation system. By reducing the lag in the angle of attack signal, the accuracy of the estimates was improved significantly. This success shows the benefits of preliminary analysis using high-fidelity simulated flight test data. This problem would have been difficult to identify with flight test data, but with the simulation-based tools it was easily isolated and corrected.

Acknowledgment

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Altitude for Maximum Angular Velocity About the Earth During Hypersonic Cruise

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Nomenclature

C_A = combined aerodynamic coefficient, $C_L + C_D \tan(\alpha - \varepsilon)$
C_D = aerodynamic drag coefficient
C_L = aerodynamic lift coefficient
g = local gravitational acceleration
m = mass of vehicle
r = radius (altitude) of flight
S = reference area
T = engine thrust
t = time
V = flight speed
α = angle of attack
γ = flight-path angle
ε = thrust direction angle relative to vehicle longitudinal axis
η = throttle setting
μ = gravitational constant
ρ = atmospheric density
φ = range angle (true anomaly)

Introduction

FOR hypersonic cruising flight there exists an altitude resulting in maximum angular velocity about a nonrotating planet. Although such an altitude does not correspond to minimum fuel consumption, it would be the ideal one at which to fly to minimize the time of flight between two given points on a planet's surface (neglecting of course the times of ascent and descent). Using the theory of ordinary maxima and minima, one can identify this altitude and the corresponding value of the maximum angular velocity by means of a very simple graphical solution. The authors stumbled upon this result while working on the longitudinal stability of hypersonic cruising flight,[1] and since they have not found it documented in the literature, they present it here as a short Note with the hope that it will be of some practical or at least of some academic or theoretical interest.

Equations of Motion

Consider the longitudinal translational motion of a high-performance aircraft, in the plane of a great circle, over a nonrotating spherical Earth. The aerodynamic plus propulsive forces acting on the vehicle during such motion can be accounted for by their components F_T and F_N that are, respectively, tangent and normal to the flight path. Explicitly,

$$F_T = T\cos(\alpha - \varepsilon) - (C_D/2)\rho S V^2 \tag{1}$$

$$F_N = T\sin(\alpha - \varepsilon) + (C_L/2)\rho S V^2 \tag{2}$$

It will prove useful to define a combined aerodynamic coefficient C_A by

$$C_A = C_L + C_D \tan(\alpha - \varepsilon) \tag{3}$$

Then, using C_A, F_N can be related to F_T via

$$F_N = F_T \tan(\alpha - \varepsilon) + (C_A/2)\rho S V^2 \tag{4}$$

Under the assumption of constant mass, the differential equations governing the motion of the center of mass of the vehicle can be written as

$$\dot{V} = (F_T/m) - g\sin\gamma \tag{5}$$

$$\dot{\gamma} = (F_N/mV) - [g - (V^2/r)](\cos\gamma/V) \tag{6}$$

$$\dot{r} = V\sin\gamma, \qquad \dot{\varphi} = V\cos\gamma/r \tag{7}$$

where the dot denotes differentiation with respect to t. It will be assumed in the remainder of this Note that 1) the flight regime is hypersonic, that is, V is very large; 2) the atmospheric density is a function only of altitude (radial distance), i.e., $\rho = \rho(r)$; 3) the local gravitational acceleration is inverse square, i.e., $g = \mu/r^2$; and 4) the thrust is an arbitrary function of the throttle setting, speed, altitude, and angle of attack of the vehicle, namely, $T = T(\eta, V, r, \alpha)$. In general, the aerodynamic lift and drag coefficients are functions of the Reynolds number, Mach number, and angle of attack of the vehicle. Because of the assumption of hypersonic regime, however, in the remainder of this Note the dependence of these coefficients on the Reynolds number and Mach number will be completely neglected.[2]

Equilibrium Solutions

Cruising flight, which will be denoted using a zero subscript, corresponds to an equilibrium solution of the fourth-order system, Eqs. (5–7), and is obtained by determining constant values of the states and controls that satisfy these differential equations. Clearly, the cruising flight path must be a great circle. Once its constant radius r_0 (corresponding to the cruise altitude) and the constant angle of attack α_0 along it are specified, the remaining steady states and controls are given by $\gamma_0 = 0$, $\dot{\varphi}_0 = V_0/r_0$, and

$$V_0 = \left[\frac{2\mu m}{2mr_0 + SC_{A0}\rho_0 r_0^2}\right]^{\frac{1}{2}} \tag{8}$$

$$T_0 = T(\eta_0, V_0, r_0, \alpha_0) = \frac{C_{D0}\rho_0 S V_0^2}{2\cos(\alpha_0 - \varepsilon_0)} \tag{9}$$

$$C_{A0} = C_{L0} + C_{D0}\tan(\alpha_0 - \varepsilon_0) \tag{10}$$

Note that it is assumed here that for given r_0, α_0, and ε_0, Eq. (9) has one and only one solution for the nominal value η_0 of the throttle

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